For office use only	Team Control Number	For office use only
T1 ________________	**65468**	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**C**	F4 ________________

2017
MCM/ICM
Summary Sheet

Summary

-Jack:oh,damn!There's another traffic jam on the high way.
-Kate:How can there be a traffic jam on a 8-lane highway every day?

Nowadays, the rapid growth of cars increase the road burden in the US. Fortunately, the emergence of self-driving,cooperating cars bring a gospel to solve the endless traffic congestion.

To demonstrate the fundamental cause of phantom traffic congestion, we simulate the congestion process. We find the slow reaction time of human drivers has a great impact on the traffic congestion. Then we establish the intelligent driver & self-driving model, which simulates human's decision in the original transportation system. This model is used to calculate the average passing time by inputting lanes, traffic density and road length to reflect the traffic effiency.

Considering the cooperation between autopilot cars, we develop an intelligent fleet model in which each autopilot can be synchronized when brake and accelerate, resulting in a stable car-following condition and reduced road Blockage. Through the further revision of the IDM model, we calculate average passing time of the autopilot vehicle. The model is also represented using a three-dimensional thermogram.Next, we use Excel data to do machine learning in the improved IDM as a input stream, and then combine it with the resulting output stream to train a random forest regression model so we can predict the output stream quickly and accurately in the future when we obtain new input stream.

Finally, we expect to build a model to help government make global optimization decisions. This model is based on RIS to obtain data shared by cars, and then transmit data to the central server using random forest model to predict the average road passing time. For each, feedback can help them in the form of local optimal planning, and for the Washington state government, they can use 3D thermal of the average passing time by controling the proportion.

Contents

1 Introduction

1.1 Background

The traffic capacity in many regions of the United States is so limited that may cause long delays during peak traffic hours. It is particularly pronounced on Interstates 5, 90, and 405, as well as State Route 520 in the state of Washington. "The estimated cumulative cost of traffic congestion by 2030 is $2.8 trillion in U.S., as many as the amount of Americans collectively payment of taxes last year", demonstrated in the INRIX reports.[1] To alleviate traffic pressure, governments generally constructing new roads but because of the limited land areas, this solution is no longer the best one. In fact, only 6 percent of roads were added between 1990 and 2010 in the United States, while the vehicle miles traveled increased by 38 percent. [2] Obviously, there is still a serious contradiction between supply and demand in urban transportation. An alternative solution is to promote self-driving, cooperating cars, which will increase capacity of highways without increasing number of lanes or roads. However, the behavior of these cars interacting with the existing traffic flow and each other is not well understood at this point.

1.2 Restatement of the Problem

To analyze and evaluate the quantitative effects on the existing traffic situation of the policy allowing self- driving ,cooperating cars in 5,90,405 Interstate Road and 520 State Road, there are four questions which we are ought to answer listed below ůHow to evaluate the traffic situation with quantitative item? ůHow to simulate existing traffic system? ůHow the item changes while adding self-driving cars to the existing system? ůHow to cooperate with other cars including self-driving cars and manual cars? Besides, we are also asked to research on how the number of lanes, the peak traffic time and the proportion of two kinds of cars influence the traffic flow. That means we have to construct a model which can link the number of lane ,the peak traffic time and the proportion to the quantitative item we build. Apart from that, we should also find the change rule of this item and determine that if there is a turning point or equilibrium point. All of this information above will be the critical basis for the state government to make policy changes.

Table 1: Notations Table

	Symbols	Meanings	Unit
Road (One Way)	S	Length of the road	km
	L	Number of lanes	
	ρ	Traffic density of the road	km^{-1}
	f	Hourly traffic count	h^{-1}
	V_0	Maximum speed limit	km/h
	α	The self-driving car proportion	
Car	s	Distance between current car and front car	m
	x	Current position of the car	m
	y	Current lane the car is on, $y = 0, 1, 2 \cdots$	
	v	Current velocity of the car	m/s
	a	Current acceleration of the car	m/s^2
	v_d	Desired Speed	km/h
	a_0	Designed maximum acceleration of the car	m/s^2
	a_1	Designed maximum deceleration of the car	m/s^2
	l	Length of the car	m
	v_f	the velocity of the car in front of current car	m/s
	v_h	the velocity of the head car of the convoy	m/s
Phantom Jam	s	Distance between current car and front car	m
	β	Acceleration factor	s^{-1}
	N	Number of cars in the chain	
	T_r	Reaction time of drivers	s
	T_d	Total delay time	s
IDSM	θ	Lane change tolerant	m/s^2
	δ	The ability of acceleartion	
	λ	Sensitivity to front car speed	
	v_h	Safe time headway	s
	s_0	Jam distance	m
	T_s	Safe time headway	s

2 Notations

3 Model One - Phantom Traffic Jam Simulation

Intersections are often considered as causes of the traffic jam, while the situation is different on the highway. Fewer intersection are in the highway and roads are relatively straight forward. Even though collisions occur on the highway everyday, accidents are not the whole reason. Generally, traffic congestion is just caused by the high load of the road. At peak time, even there is neither any accident nor enclosure construction, traffic jam happens from time to time. Everything is fun but the travel time is long. This phenomenon is called Phantom Traffic Jam.

3.1 Assumptions

- **The traffic count are evenly distributed by lane.** The increasing direction and decreasing direction of one road share traffic volume based on the ratio of number of lane.
- **The vehicles on the road are same except for the desired speed.** The cars has same length, designed acceleration and max deceleration etc. But each car have different desired speed.

3.2 Establishing the Model

The formula above describe the road crowdedness. f is the hourly traffic count obtained by dividing the daily traffic count data from excel spreadsheet into 24 partition. Figure 1 shows two different road situation with different ρ.

$$\rho = \frac{f}{V_0} \tag{1}$$

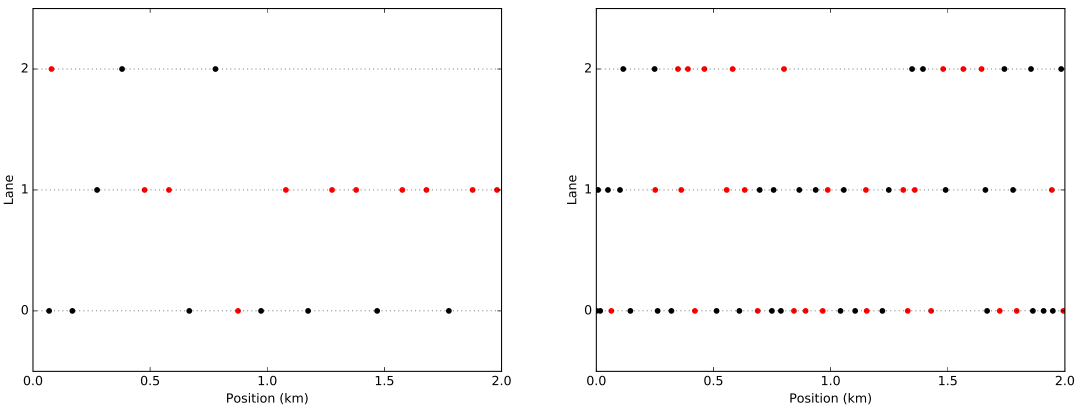


Figure 1: The different road traffic density simulation, $\rho = 10$ (left) and $\rho = 30$ (right).

To simulate the car motion, a force(or acceleration) need to be applied to the car. We introduce an heuristic formula to calculate the acceleration at each update frame:

$$v(s) = \begin{cases} \beta s & , s < \frac{V_0}{\beta} \\ V_0 & , s \geq \frac{V_0}{\beta} \end{cases} \tag{2}$$

3.3 Simulation

We use the data given by the excel spreadsheet to build the road model, and then simulate the car movement on the road. Random noise are applied to the velocity of each car to simulate the response time of drivers differs by the weather, road conditions, physical conditions, etc.

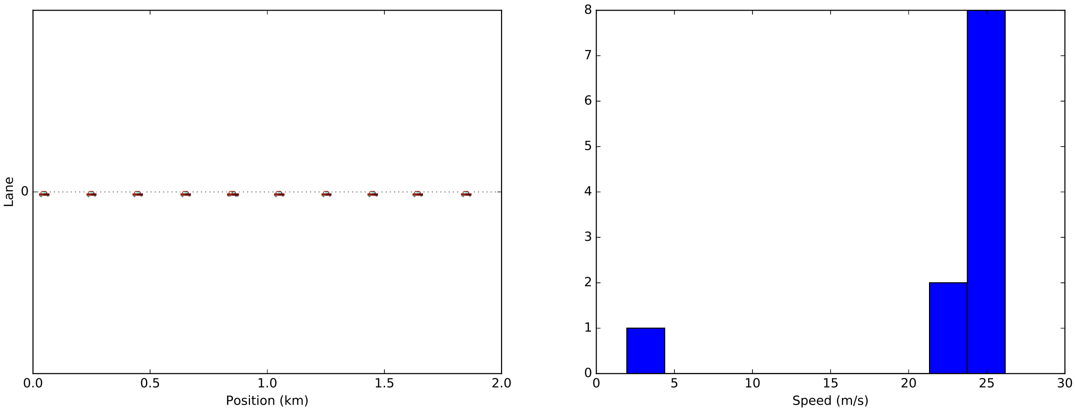


Figure 2: Stable running situation, no phantom jam.

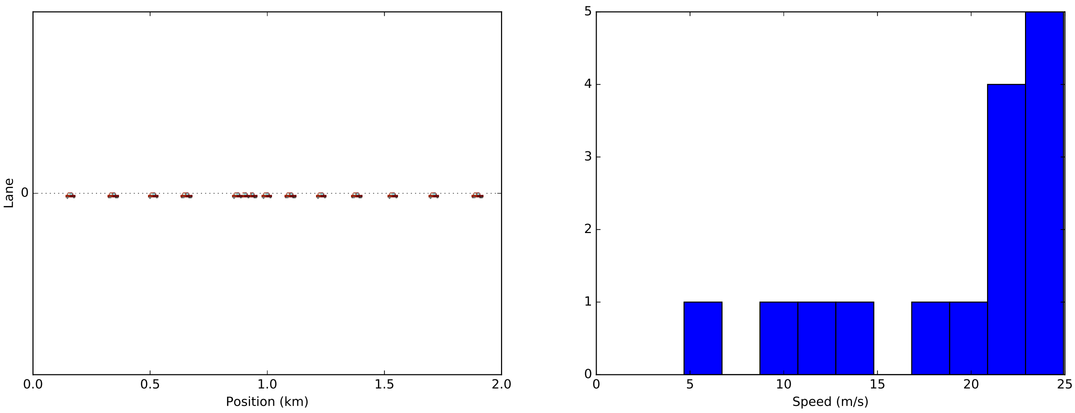


Figure 3: Phantom jam occurs.

The second figure in above demonstrates a phantom jam situation. It caused by the brake of the first driver, which triggers the drivers behind brake one by one.

$$T_d = NT_r \tag{3}$$

Similarly, drivers restart one by one, expanding the bad influence again. The whole process we describe above cost vast time and finally will lead to a more serious congestion.

By analysis we can learn that the non-synchronicity of information contributes to the delay of traffic jam, i.e., the last driver in the queue cant get the information of the first driver has braked at the first time. Assuming that the transmission of information can be non-time-consuming, that is, each driver in the traffic queue can get information and take the same action at the same time, then the problem of phantom traffic jam has been solved.

The study of this problem has made us wonder the differences between self-driving cars and manual driving cars. The self-driving cars can greatly reduce the information transmission time and is expected to eliminate random slow-down process of each driver of manual driving cars. Therefore, the study of self-driving, cooperating cars may help to ease or even solve the traffic jam.

4 Model Two - Intelligent Driver and Self-driving System

Under the shroud of phantom traffic jam, vehicles seems likely to be trapped when road density is high. Traditional cars generally has a long response time and unstable speed, which may be the reason of traffic congestion. We build the IDSM(Intelligent Driver and Self-driving model) based on the Intelligent Driver Model [3], which is used to evaluate the self-driving vehicle's influence to the current situation.

4.1 Assumptions

The following assumptions are listed to simplify the model.

- **Left overtaking**. According to the law of Washington state, drivers must drive in the right-most lane and overtake the left. So only left direction overtaking are considered in this model.
- **Neither the traffic situation at highway entrance, exit ramp, traffic lights nor accident.** Phantom traffic phenomenon are considered as the only way to cause the highway traffic jam in this model.
- **Neither HOV nor Carpool lane.** For the open time of the fast lane are short, they are not considered in the model.
- **Smart car is same with traditional car on hardware** That is, they have the same parameters of models such as power, length, width. The length of cars are the standard 5 meters and the largest passengers number is 5.

4.2 Establishing the Model

We use the road simulation model developed in model one to build our second model. Additionally, we divided the vehicle into two types, self-driving and traditional vehicle. The behavior of an individual vehicle, whether it is a self-driving vehicle or a manual driving vehicle, can be described in the formula below.

$$a(s, v, v_f, \Delta v) = a_0 \left[1 - s^*(v_f, v) \left((\frac{v}{v_0})^\delta + b^*(s, v, \Delta v) \right) \right] \tag{4}$$

and $b^*(s, v, \Delta v)$:

$$b^*(s, v, \Delta v) = \left[\frac{(s_0 + Tv + \frac{v\Delta v}{2\sqrt{a_0 a_1}})}{s} \right]^2 \tag{5}$$

where $s^*(v_f, v)$:

$$s^*(v_f, v) = (\frac{v}{v_f})^\lambda \tag{6}$$

Brake term $b^*(s, v, \Delta v)$ is used to provide deceleration for car simulation. When the distance between current car and front car is too small, this term will force the car to stop by decelerating the car. We define overtaking endurance threshold as θ. When the threshold level is exceeded, overtaking happens.

Formula 6 factor is the front-car-velocity sensitive, which is a factor that specifically reflects the self-driving car characteristic. As we known, the self-driving cars have the camera, radar sensors and if the front car is also a self-driving car, the can even communicate with each other to send the motion state information. So, different from traditional car, self-driving car has a better capability to sense the front car, which means it has a higher λ value, λ is one of important factors to distinguish self-driving car to manual driving car.

In addition to the motion advantages, smart car, comparison to traditional car, has another advantage, which is the convoy strategy. As the smart car can easily now the motion state(velocity acceleration etc.) of the front smart car, then it can obtain the state of the front smart car of the front smart car too! In other words, smart cars can line up on the highway and take orders from the head car. This help reduce T_d and improve the traffic efficiency.

The lane change and convoy follows rules below.

- All lane changes must start from a higher lane number and end at a lower lane number.
- A head car is the first car in a convoy.
- If there is a front car of the head car in this road section, it must be a traditional car.

- If a smart car is in a convoy, it can quit the convoy only when the front car is not a smart car.

Algorithm 1 Lane Change

Require: θ, v, x, y, d
  if $b^*(d, v, \Delta v) < \theta$ and no car on the $y-1$ lane is within the distance of vT **then**
    car k move to $y-1$ lane.
  else
    brake.
  end if

4.3 Parameters

Other parameters with their values are given in the table below

4.4 Simulation

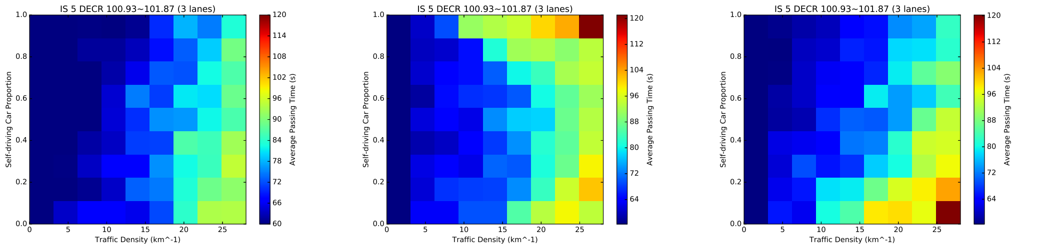
Three different strategy are used in the simulation.

1. **No reserved**. No lane is reserved for smart cars.
2. **Reserved with limitation**. The 0th lane is reserved for smart cars. Smart car can run and only run on this lane.
3. **Reserved without limitation**. The 0th lane is reserved for smart cars. Smart car can run on whichever lane it likes.

According to the figure 8. We can find that the traffic achieves the best result when no special lane are reserved for the self-driving car. The maximum average

Table 2: parameters

Parameters	Self-driving car	Traditional car	Unit
s_0	2	2	m
T_s	4	4	s
v_d	26.82	$\sim N(26.82, 2.68)$	m/s
a_0	0.8	0.8	m/s^2
a_1	1.8	1.8	m/s^2
l	5	5	m
θ	0.5	0.5	
δ	4	4	
λ	2	0	



(a) No reserve (b) Reserved with limitation (c) Reserved without limitation

figure 4: Random forest prediction

passing time is about 96, and it shows that more self-driving car the better. When a special lane is reserved for self-driving car but the smart car can't move to other lanes, the system didn't perform any better than the first situation, and come to the worst point when all the car on the road is self-driving car. The third one reserves a lane for self-driving cars, and the the self-driving cars can burst into the traditional cars' lane, and results show it performs better than the second situation and demonstrates the positive effect of self-driving car on the mixed lane situation. But it still perform no better than the non-reserved situation.

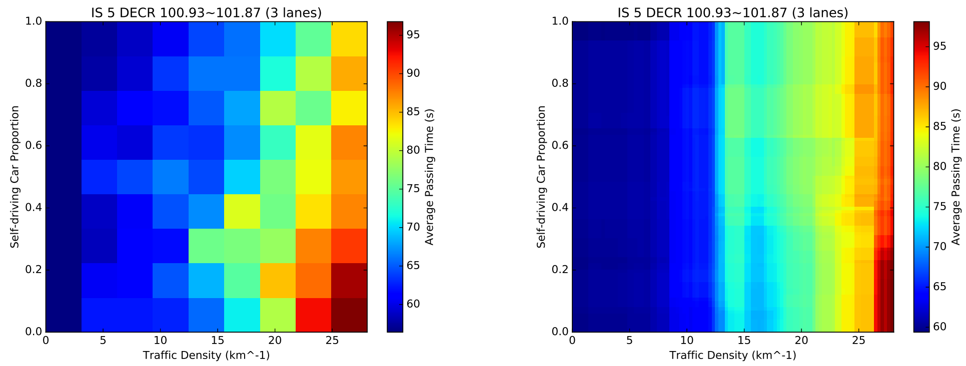
4.5 Regression

Generating the heat map with a high sample rate for all the road sections in the given excel spreadsheet means huge amount of calculation. It's hard to simulate all the situations with the limited calculation resources. We use Random Forest Regression model here to fit the simulation result.Random Forest Regression model is a tree based regression model, which has great generalization ability [2].

To use the model, We choose some densities based on the daily traffic count and pick the self-driving car proportions randomly. Finally we get 17920 records after 6 hours' simulation. Data are finally separated into two part, train part and valid part, whose ratio is 3 : 1. After the model is trained, the validation is performed. It shows there are only 4.6 seconds error, which is about 5% different from the simulation result.

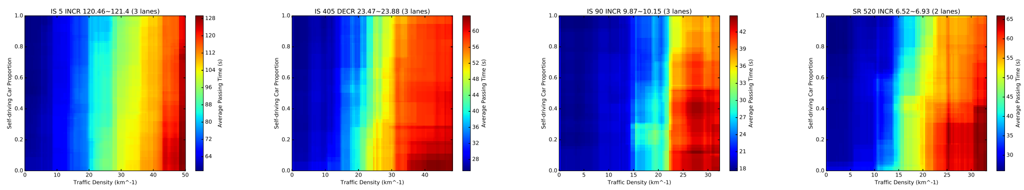
The figure 5 shows the regression result after the 6 hours training. The both of the two figure show high density brings congestion, and the self-driving car can help improve the traffic condition when the congestion occurs.

After the model is trained, we use it to predict the traffic situation of specific road under different road traffic density and self-deriving car proportions. 4 heat maps shows different effect intensity of self-driving car proportion. For example, (a) IS 5 INCR start at 120.46 seems not benefit much from the high self-driving car proportion, while (d) SR 520 INCR start 6.52, on the contrary, benefits a lot. The off-line training shows that the longer the road is, the less it benefit from



(a) Simulation (b) Random Forest

figure 5: Comparison between the simulation result and random forest result. The latter has a higher resolution



(a) IS 5 INCR 120.46 (b) IS 405 DECR 23.47 (c) IS 90 INCR 9.87 (d) SR 520 INCR 6.52

figure 6: Random forest prediction

high self-driving car.

5 Model-Three Cooperating Central Control Model Based On the Route Information system

To evaluate traffic condition, we introduce a macroscopic index named traffic efficiency because all factors cause traffic congestion by bringing down traffic efficiency. When a certain factor is acting on a traffic flow, the increase of traffic

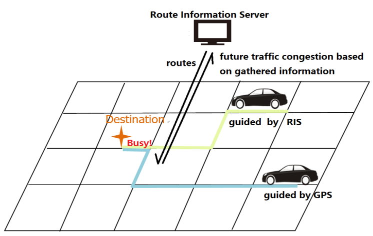


figure 7: time T

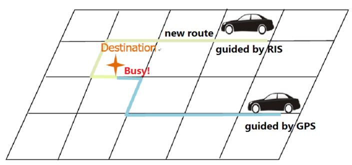


figure 8: time T+1

efficiency indicates that some factors have a positive impact on traffic flow while the decrease indicates a negative impact. We have defined as the time needed for a single car to pass through a block at the beginning of the paper. We will make a more detailed description of below because the understanding of it can help to make better decisions of route.

Firstly, the ratio of auto-driving to manual driving have an impact. We introduce ET and to describe traffic efficiency and introduce Eh and Dh to describe road congestion degree. For the transportation system, the smaller ET, the higher the efficiency and the better the traffic condition. the lower the road congestion degree is. Ratio of auto-driving to manual driving on the road can affect traffic situation by changing ET because once the new selection be acted, the transit time of all autopilot vehicle, the average, the variance of transit time of all autopilot vehicles will change.

Secondly, the number of lanes and the different time periods are also factors influencing . Although they do not appear directly in the form of variables in 3D surfaces, they affect the transit time indirectly by influencing the independent variables in the 3D surface:

Consider the effect of different time periods. We determine the whole peak time last for two and a half hours by survey and know the fact given in the problem that 8% of the daily traffic volume occurs during peak travel hours, so the parameters of vehicle density can be given directly. Knowing the vehicle density, we can look up the table and get the corresponding time value and then get different effects of peak hours and daily time on when the autopilot and non-autopilot vehicle ratio stay unchanged. The figure is shown below:

Consider the effect of the number of lanes. Because of the constant value of traffic flow per unit time, the more traffic lanes, the lower the traffic density is.

5.1 Establishing the Model

Currently, the two main self-driving cars are produced by Google and Tesla. Google advocates the use of algorithms while Tesla is more advocating the use of data to optimize route selection. In this model, the focus of our study is using data to optimize route selection.

We first built a pure manual driving cars system, which have no self-driving car. Its decision-making path is selected by GPS based on time and distance. Then we get a cooperative system formed by adding the automatic driving to the existing traffic flow under different proportions. Cooperative system can be divided into two levels, one is the collaboration between autopilots and manual driving cars and the other is collaboration between auto-cars. Well discuss the cooperative process between autopilot and non-autopilot first.

There are two general methods to consider the route-selection of a single car. One is the shortest path method, which can save fuel costs but may increase the cost of time. The other is shortest time method, which can save time costs but may increase the cost of fuel. Our team will use a new method that take both distance saving and time saving into account, that is, Route Sharing System (RIS) method.

The process of RIS is that manual driving cars and self-driving cars submit their passed path to RIS, after processing an optimal centralized system to obtain the comprehensive consideration of the shortest time and the shortest path solution path provides a smart car to follow, and artificial driving advice

Nowadays, most vehicles were navigated by satellites in the GPS (Global Positioning System), but the route-decision of each driver in the same transportation network is unknown to each other. This "Information Blind Spot" leads to a situation of all choosing the same road and thus this road is packed with cars and traffic congestion happens.

To solve the problem mentioned above, we expect all cars to share their data such as position, velocity, destination and route for analysis and will provide better travel options for everyone. In view of passengers may concern about the security problem caused by leakage of private data, we build CCCM (cooperating central control) model based on RIS to collect information from the global perspective and achieve central optimization.

In CCCM, self-driving cars will provide more information than ordinary manual driving cars. It will use its own data acquisition device to collect more data such as accurate real-time road conditions, three-dimensional model of traffic flow, position, velocity destination and route of themselves and the vehicles around them. At the same time, its V2X technology will make the timely communication with other cars on the road come true and improve the accuracy of information collecting. Then, they will share the information to one or more central control server. After analysis, central control sever will fed back to cooperating cars to navigate and provide the user with the best path choice.

The two most important parameters in CCCM is that d and s. These two parameters will directly determine and can form a 3D surfaces with . Since drivers in traditional car dont have the value of d and the ratio of manual cars to self-driving cars, they choose route only based on calculated by GPS, which is general static while self-driving cars can look up table to get the modified time after selecting destination. With the change of the traffic condition, the traffic density of a single road and the ratio of manual cars to self-driving cars will change and the table will be refreshed, and a new route will be chosen.

The key of the model is to make the relationship table between and d, s. There are two ways to make it, one is using microscopic IDM model and the other called Random Forest. IDM model is a discrete model, so we cannot directly derive the functional relationship between and P,D. In this model, we change the initial parameters of the model according to the actual situation:

Another way is using the random forest model, which constantly adding real-time data to train to predict new . Obviously, using random forest is more convenient, but because of the very strong micro-thinking IDM has, we still consider it as a good method.

After obtaining sets by two methods mentioned above, we will use dynamic programming to iterate to solve the optimal path.

Based on the analysis above, we find that Random Forest and RIS system can provide different services for different roles. For traffic management,

6 Policy changes in Washington State

- Allow self-driving cars but control the percentage of self-driving cars;
- Do not set up dedicated lanes;
- Invest on building self-driving car convoy and Route Information system;

7 Strengths and weaknesses

7.1 Strengths

- **Applies widely**
  Our model is widely applicable.The data in CCCM can be used by different people to make different choices.For example ,CCCM can provide local programming optimization for individual drivers and global optimization for the manager or officer.
- **Use a machine learning algorithm**
  We use a machine learning algorithm –random forest to optimize our computing process.This algorithm increase the speed and accuracy of predicting data.
- **Make full use of the data**
  We have made full use of the data from the Excel provided. All the data produced in our different models originate from Excel table.
- **Ponder over the cooperation from two levels**
  We ponder over the cooperation from two levels: the first level is the micro level–the coordination between self driving,cooperating cars;the second level–the shared data
- **Models are interrelated**
  Our models are interrelated,because each model in our essay is relevant to

the basic IDM .We improve the original IDM model and adjust its parameters according to the self-driving cars which make it better suited to self driving cars.

- **Assumptions are reasonable**
  Our Assumptions are rather reasonable,because all of them are certified in practical statistics.We even refer to the Washington State Law to make sure the assumptions are applicable to the reality

7.2 weakness

- **Lack of data**
  Random forest training sample data is not large enough. Due to the limited number of autopilot cars, autopilot can only obtain part of the location, route information, so the decision-making is only local optimization.
- **Regardless of people'mind**
  The optimal route planned by the central control system can be adopted by the autopilot vehicle, but the optimal path suggestion shared with the driver may not be adopted.

8 Letter to Government

Letter to the governor of Washington State Dear sir: Upon hearing that you are considering to allow self-driving ,cooperating cars on the roads in Thurston, Pierce, King, and Snohomish counties to solve the annoying traffic jam problem, we are honored to provide some ideas and recommendations. We approve of this policy and are convinced that it will make contribution to decline the traffic congestion. I will elaborate and explain our recent research on self-driving, cooperating cars interacting with the existing traffic system, which may help you formulate the policy. From our analysis, there are three reasons for allowing automatic driving. Firstly,when there is no accidents or obstacles, there are still always traffic jam on the interstate highway and the state road in Washington, which called "phantom traffic jam". An important way to solve this problem is to ensure the synchronization of the driver's operation. Besides, when the density is a constant, with the increase of the proportion of self-driving car, the average travel time become shorter, which indicates the efficiency of the road becomes higher. Secondly, in order to prove this quantitatively, we simulated out of the existing traffic network, using average road travel time to reflect the road graphics efficiency. When we add the autopilot to it, we find that the traffic network has been improved obviously. This three-dimensional thermal chart can show this.

Thirdly, though adding self-driving car is beneficial for reducing the traffic congestion, it not appropriate to improve the proportion excessively. From this

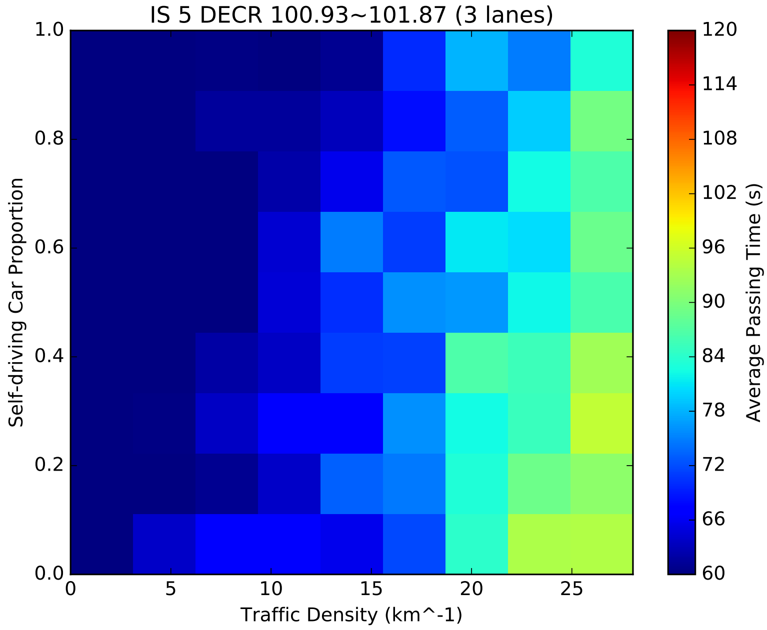
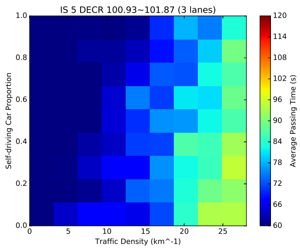
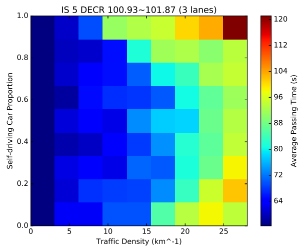


figure 9: No special reseverd Lane

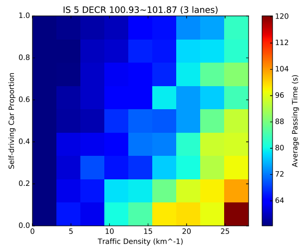
chart, we find that when the proportion goes beyond 25%, the average passing time become higher, so a possible turning point is at 25% percentage. Besides, the equilibria do exist. The average passing time will fluctuate from 84s to 94s after the sharp increase.



(a) No reserve



(b) Reserved with limitation



(c) Reserved without limitation

Through further analysis, we are not for the proposal that we should open a dedicated lane for the self-driving cars. We consider about two kinds of situations. The first one is that the automatic car can only travel on the dedicated lane , and the other one is that it can change into ordinary lane while other cars can not change to the dedicated lanes. Compared the two cases to no dedicated lane, we find that the average traffic time of the two cases is both larger than that in no dedicated line. Besides, the average passing time of automatic car only travelling on the dedicated line is largest among these cases. Under this condition, when the density is a constant and the average traffic time becomes larger and when the proportion goes up to 100%, reached the maximum average traffic time with the increase of proportion of automatic driving car. At last, there are my answer of your questions: 1.If the lanes are a few, the more lanes, the better the traffic; but too much lanes also cause the too much overtaking, which may lead to traffic jam or even accident. So three lanes may be a best choice. 2.The peak time of a day has a higher traffic density .From the thermal graphic, we can find that with the traffic density increase, the average passing time increase. 3.There is no linear relationship between proportion of self-driving cars and average passing time. Generally, with the proportion increase, the average passing time decrease. However, turning point do exists and they are relevant to the characteristic of road. If the proportion goes beyond this point, the average passing time begins to increase. One explanation for the third result is that self-driving cars are so careful that they travel very slowly even if they could have arrived at a higher speed. This can also lead to low effiency. If you want to know more detailed analysis and the data, welcome to read our paper. We wish the traffic jams in Washington can be resolved soom

Yours sincerely.

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Appendices

Appendix A First appendix

Appendix B Second appendix